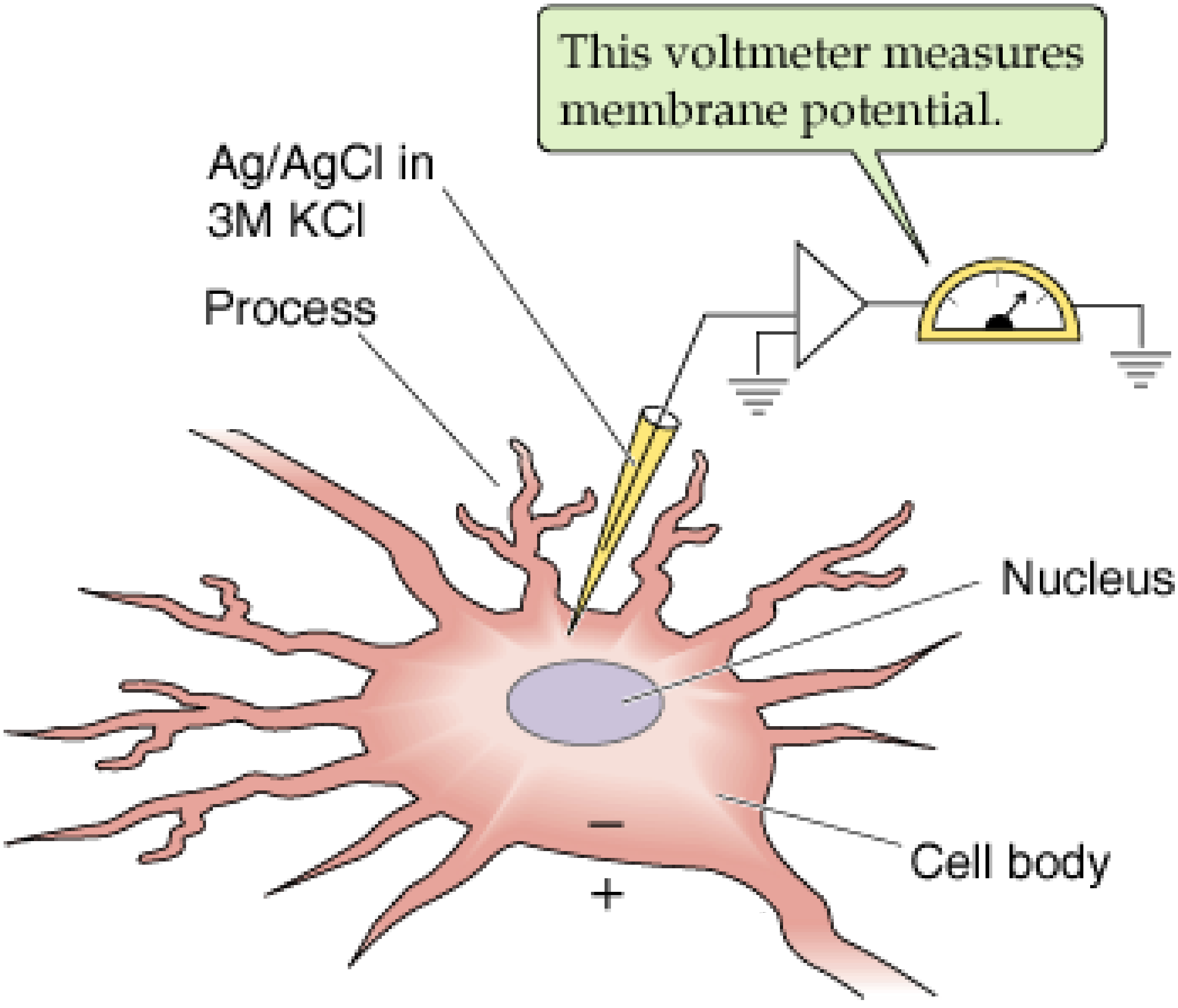
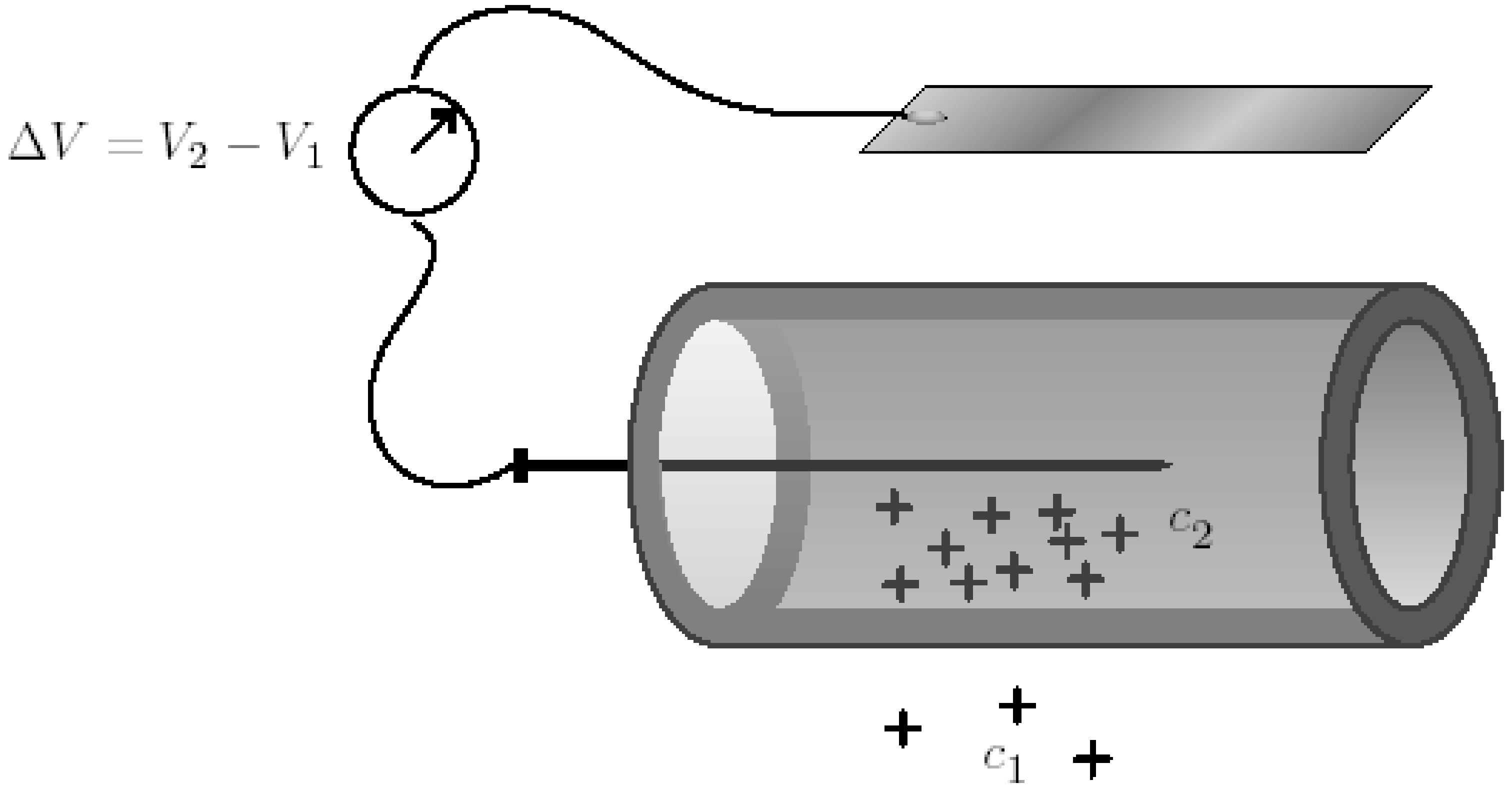
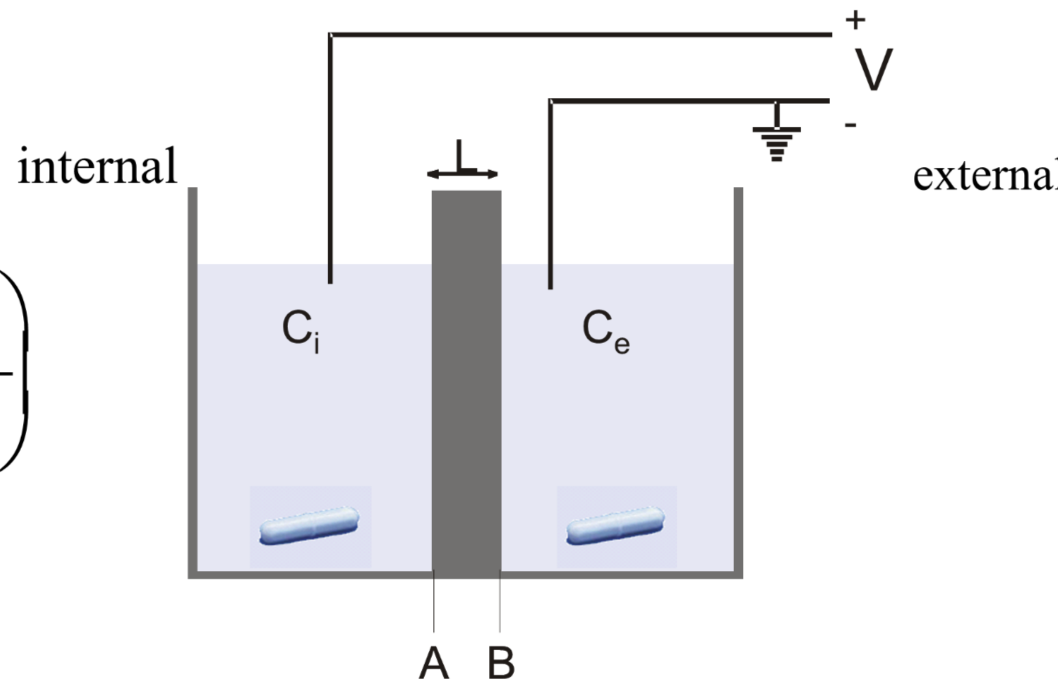

This voltmeter measures membrane potential.
Ag/AgCl in 3M KCl
Process
Nucleus
Cell body
–
+



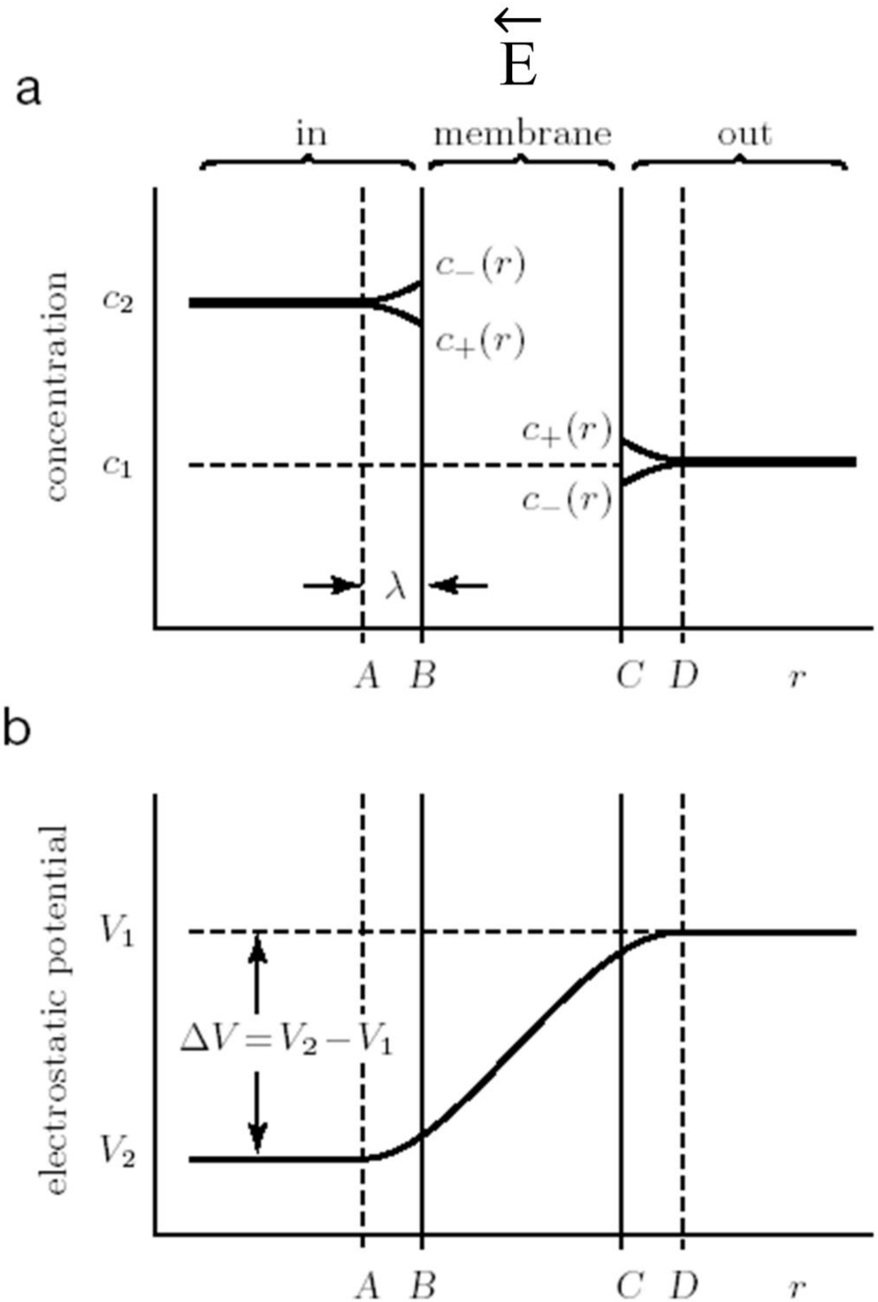
- Selective permeability to ions leads to membrane charge. Start with K^+-permeant membrane, with abundance of K^+ inside the cell.

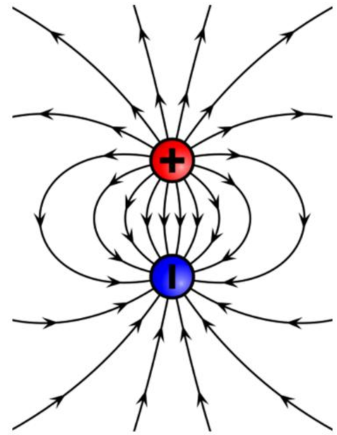
Electrodiffusion – Nernst potential

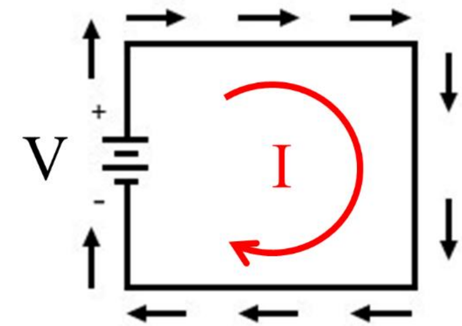
$$V^{Nernst} \equiv -\frac{k_B T}{q} \ln\left(\frac{C_i}{C_e}\right)$$



Ion	concentration (mM)		
	interstitial space	cell ("typical")	V^{nernst} (mV) (37°C)
K^+,	4.5	120	-85







- Electric fields
  - Caused by charges
  - Indicate the direction that a positive charge would move
- Electric potentials
  - Integrates field

$$V = -\int E \cdot dl$$

Nernst potentials for multiple ions

Ion	concentration (mM)		
	interstitial space (ext)	cell ("typical") (int)	V^{nernst} (mV)
Na^+, mammalian cell	145	15	+58
K^+, mammalian cell	4.5	120	-84
Cl^-, mammalian cell	116	20	-45
Na^+, squid giant axon	440	50	+54
K^+, squid giant axon	20	400	-75
Cl^-, squid giant axon	560	52	-59

Gibbs-Donnan equilibrium

- Concentrations adjust such that Nernst Potentials are identical for ALL species:
  - Since external pool is huge, internal concentrations adjust

- Electroneutrality both inside and outside
  - External: $[Na^+]_e+[K^+]_e+[Cl^-]_e=145+5+(-1)*116$ mM

    = 34 mM excess. Not bad….
  - Internal: these will be calculated, but from the standard concentrations presented earlier, there is an excess of positive charges, if only K^+, Na^+, and Cl^- are considered. 125 mM of negative charge comes from DNA/Proteins

Gibbs-Donnan equilibrium

$$V^{Nernst} \equiv -\frac{k_B T}{q} \ln\left(\frac{C2}{C1}\right) \qquad \frac{[\mathrm{Na}^+]_e}{[\mathrm{Na}^+]_i} = \frac{[\mathrm{K}^+]_e}{[\mathrm{K}^+]_i} = \frac{[\mathrm{Cl}^-]_i}{[\mathrm{Cl}^-]_e}$$

$$[\mathrm{Na}^+]_i + [\mathrm{K}^+]_i - [\mathrm{Cl}^-]_i - 125\mathrm{mM} = 0$$

Given external concentrations,
$[Na^+]_i$= 145mM, $[K^+]_i$= 5 mM, $[Cl^-]_i$=116 mM

Solving for internal concentrations,
$[Na^+]_i$= 202 mM, $[K^+]_i$= 6.3 mM, $[Cl^-]_i$=83 mM, V_R=-9 mV

Goldman-Hodgkin-Katz voltage equation

- Resting: net flow of *charge* = 0

$$j = \frac{D}{L}\frac{qV}{k_BT}\frac{C_i - C_e \exp\left(-\frac{qV}{k_BT}\right)}{1-\exp\left(-\frac{qV}{k_BT}\right)} = P\frac{qV}{k_BT}\frac{C_i - C_e \exp\left(-\frac{qV}{k_BT}\right)}{1-\exp\left(-\frac{qV}{k_BT}\right)}$$

- P (permeability) can also include partition coefficient, structure, etc.
- P_{ion} encompasses many factors for that ion

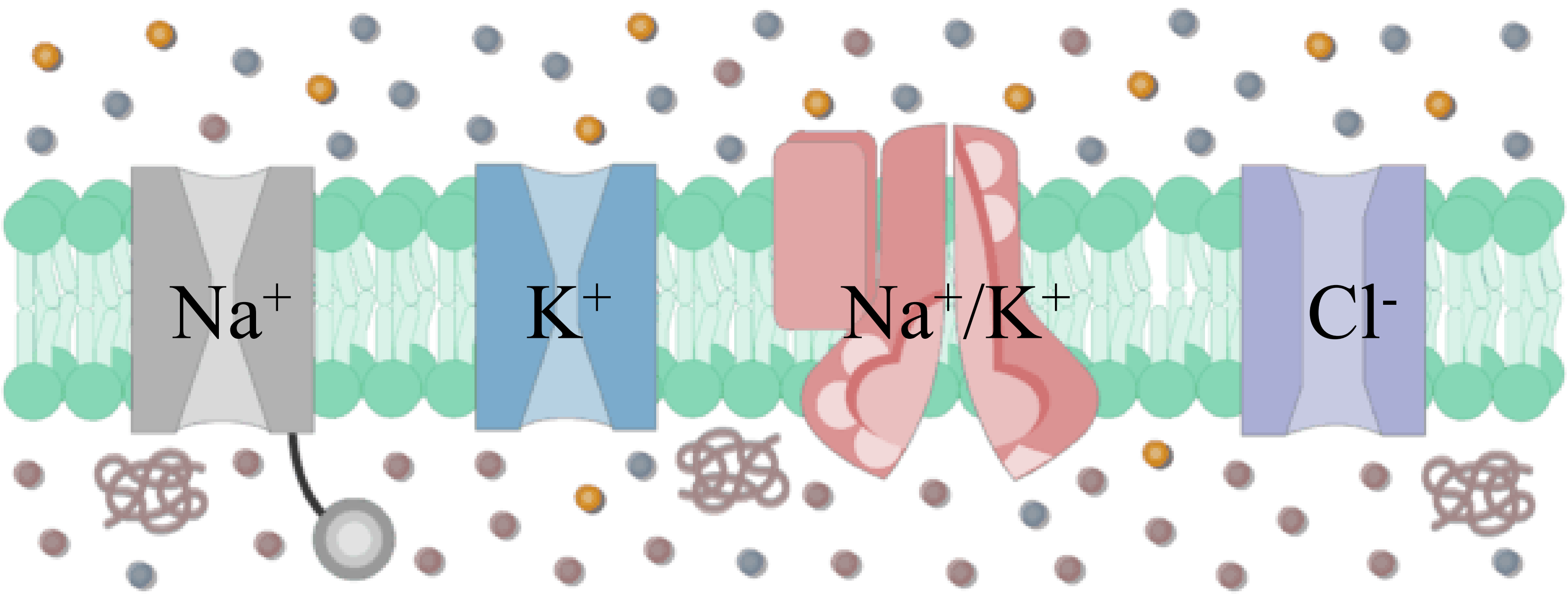
$$V_R = -\frac{RT}{F}\ln\left(\frac{P_{Na}[Na]_i + P_K[K]_i + P_{Cl}[Cl]_e}{P_{Na}[Na]_e + P_K[K]_e + P_{Cl}[Cl]_i}\right)$$

$P_K = 25\ P_{Na} = 2\ P_{Cl}$; This gives V_R=-56 mV.

Pump-leak model

- 1951: Ussing and Zehran
  Na^+ pumping by frog skin, even when $\Delta V=0$ and Na^+ conc. same on both sides.
  Na^+ is actively pumped.
- 1955: Hodgkin and Keynes showed that K^+ is also pumped, and is needed for Na^+ transport.
  Metabolic inhibitors interrupted Na/K pumping.
- 1957: Skou isolated a Na/K transporter, an active transporter, sodium pump.
  Later studies: flux of K^+ was 2/3 that of Na^+

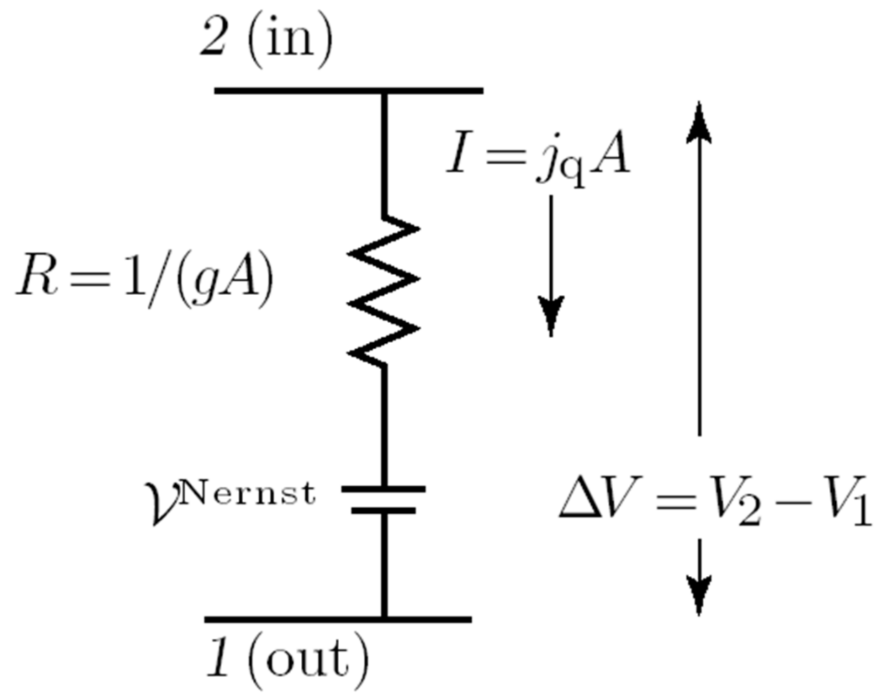
Pump-leak model



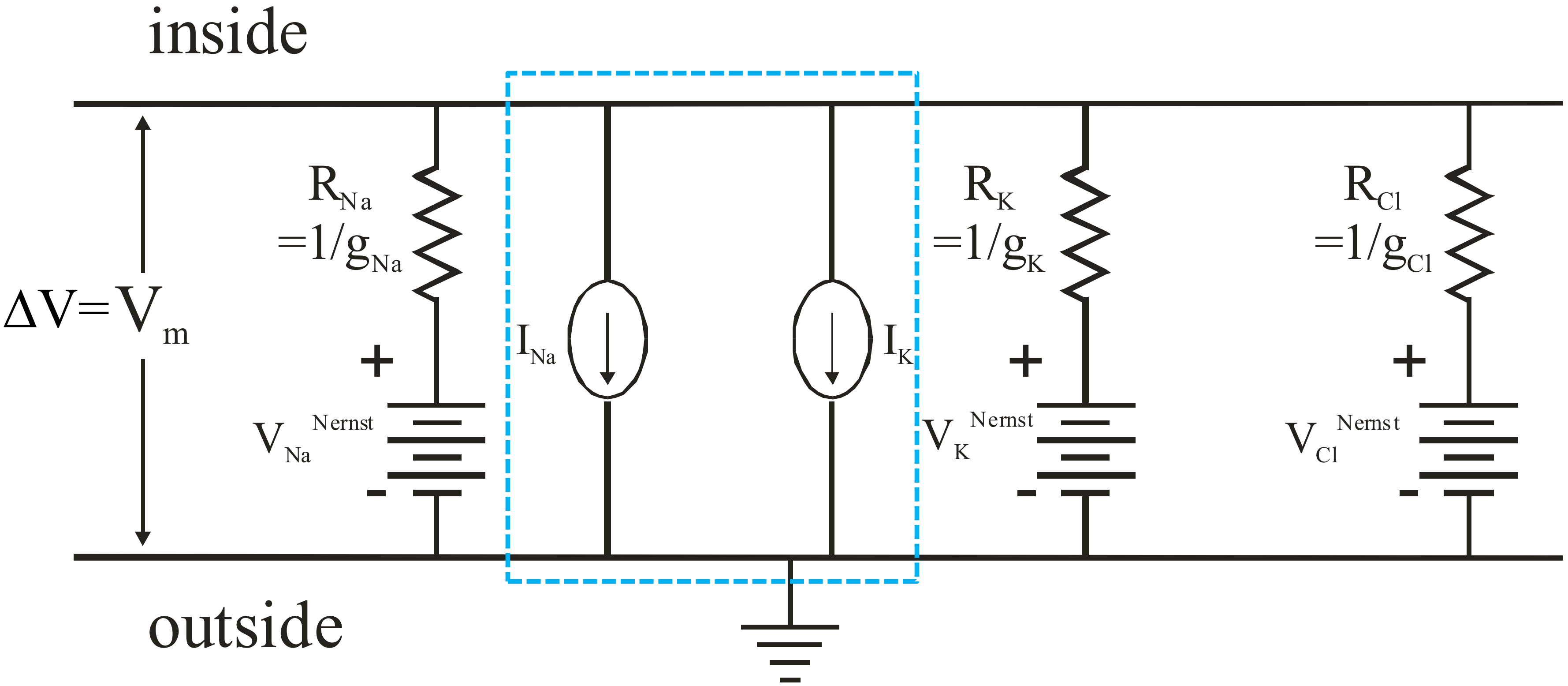
Pump-leak: leak (ohmic hypothesis)

$$j_{q,i}^{ohmic} = n_i q j_i^{ohmic} = \left(\Delta V - V^{Nernst}\right) g_i$$

$j_{q,i}$=charge flux of species i
j_i=concentration flux of species i
n_i=number of charges for species
q=fundamental charge of electron
g_i=conductance per area



Pump-leak; Na/K pump



Pump-leak; Na/K

$$j_{Na^+} = \frac{g_{Na^+}}{q}\left(\Delta V - V_{Na^+}{}^{Nernst}\right) + j_{Na^+}^{pump} = j_{Na^+}^{ohmic} + j_{Na^+}^{pump}$$

$$j_{K^+} = \frac{g_{K^+}}{q}\left(\Delta V - V_{K^+}{}^{Nernst}\right) + j_{K^+}^{pump} = j_{K^+}^{ohmic} + j_{K^+}^{pump}$$

$$j_{Cl^-} = \frac{g_{Cl^-}}{q}\left(\Delta V - V_{Cl^-}{}^{Nernst}\right)$$

Resting potential: set the flux of each species to zero

Pump-leak; Na/K

$$j_{Na^+} = \frac{g_{Na^+}}{q}\left(\Delta V - V_{Na^+}{}^{Nernst}\right) + j_{Na^+}^{pump} = j_{Na^+}^{ohmic} + j_{Na^+}^{pump}$$

$$j_{K^+} = \frac{g_{K^+}}{q}\left(\Delta V - V_{K^+}{}^{Nernst}\right) + j_{K^+}^{pump} = j_{K^+}^{ohmic} + j_{K^+}^{pump}$$

from knowledge of Na/K pump

$$j_{Na^+}^{pump} = -\frac{3}{2} j_{K^+}^{pump}$$

$$-j_{Na^+}^{ohmic} = j_{Na^+}^{pump} = -\frac{3}{2} j_{K^+}^{pump} = \frac{3}{2} j_{K^+}^{ohmic}$$

$$\left(\Delta V - V_{Na^+}^{Nernst}\right) g_{Na^+} = -\frac{3}{2}\left(\Delta V - V_{K^+}^{Nernst}\right) g_{K^+}$$

rearrange

$$\Delta V\left(2g_{Na^+} + 3g_{K^+}\right) = 3V_{K^+}^{Nernst} g_{K^+} + 2V_{Na^+}^{Nernst} g_{Na^+}$$

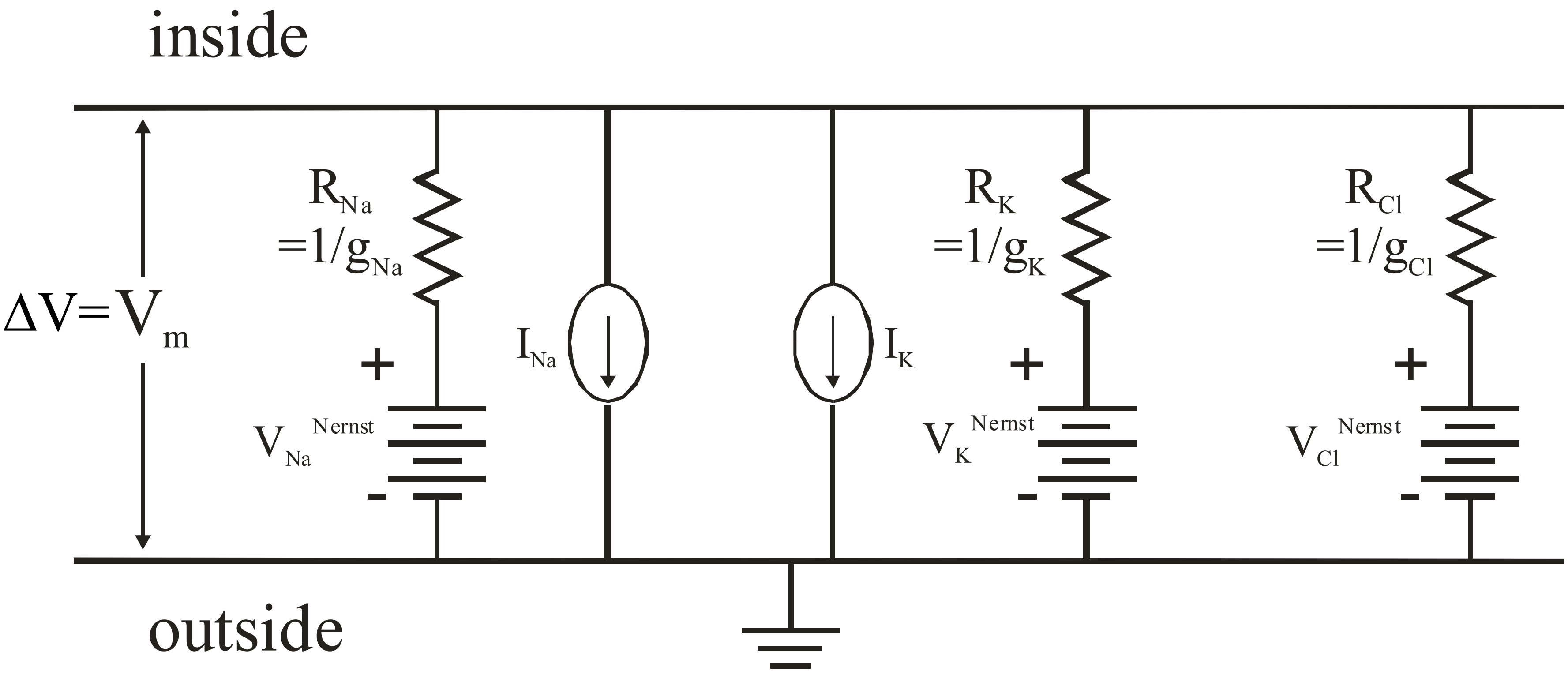
$$\Delta V = \frac{3V_{K^+}^{Nernst} g_{K^+} + 2V_{Na^+}^{Nernst} g_{Na^+}}{\left(2g_{Na^+} + 3g_{K^+}\right)}$$

for:

$$g_{K^+} \approx 25 g_{Na^+}$$

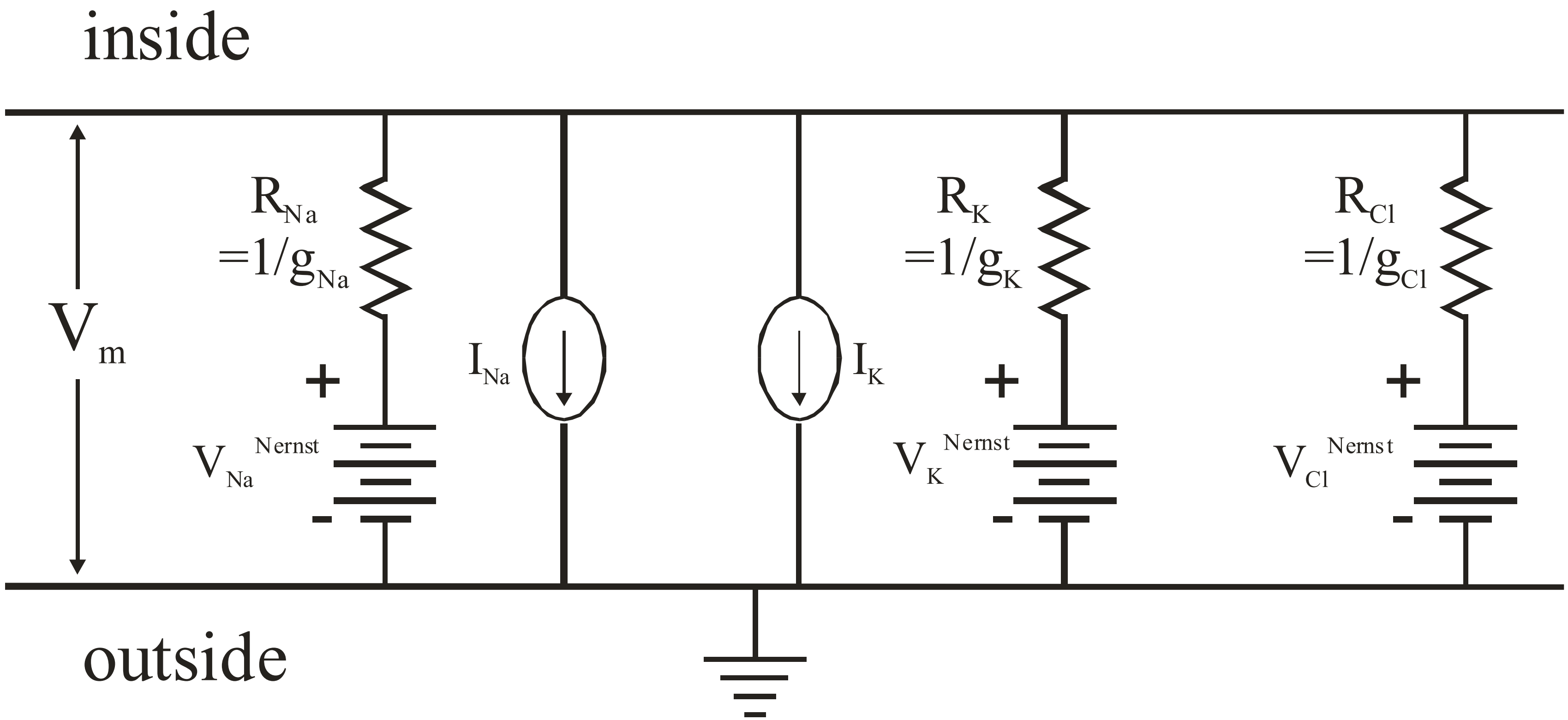
resting potentials:
mammalian: -80 mV
squid: -72 mV

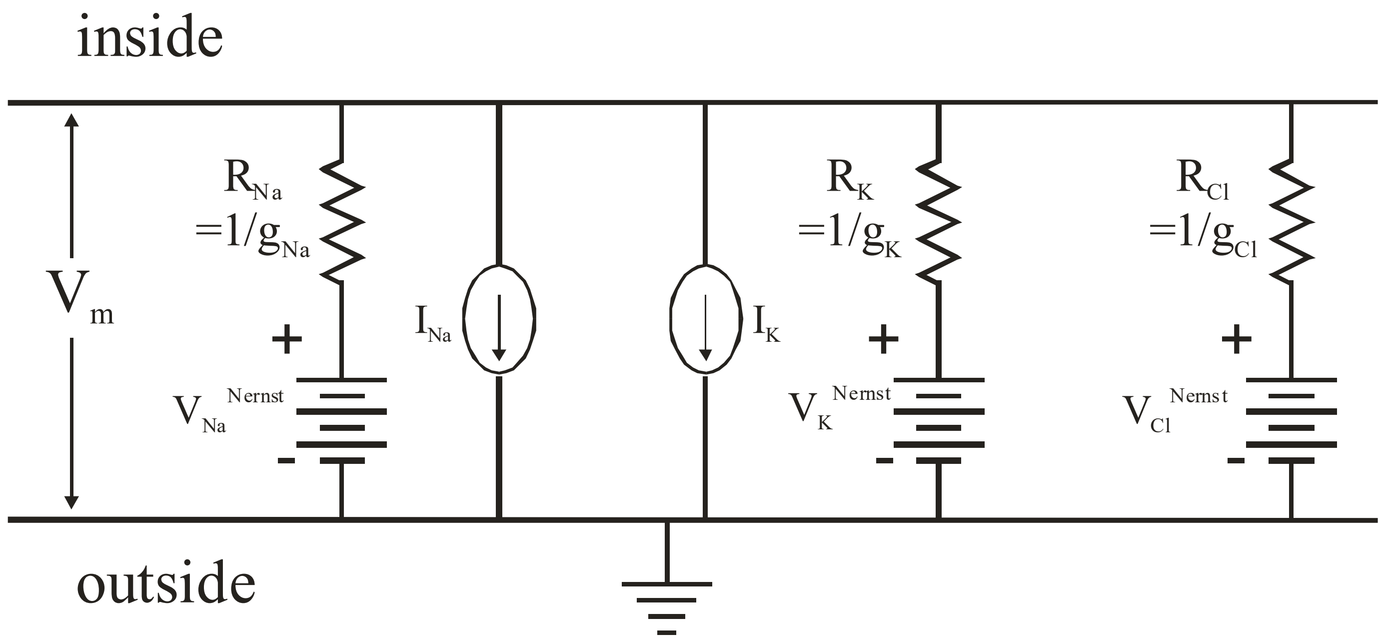
Pump-leak; Na/K

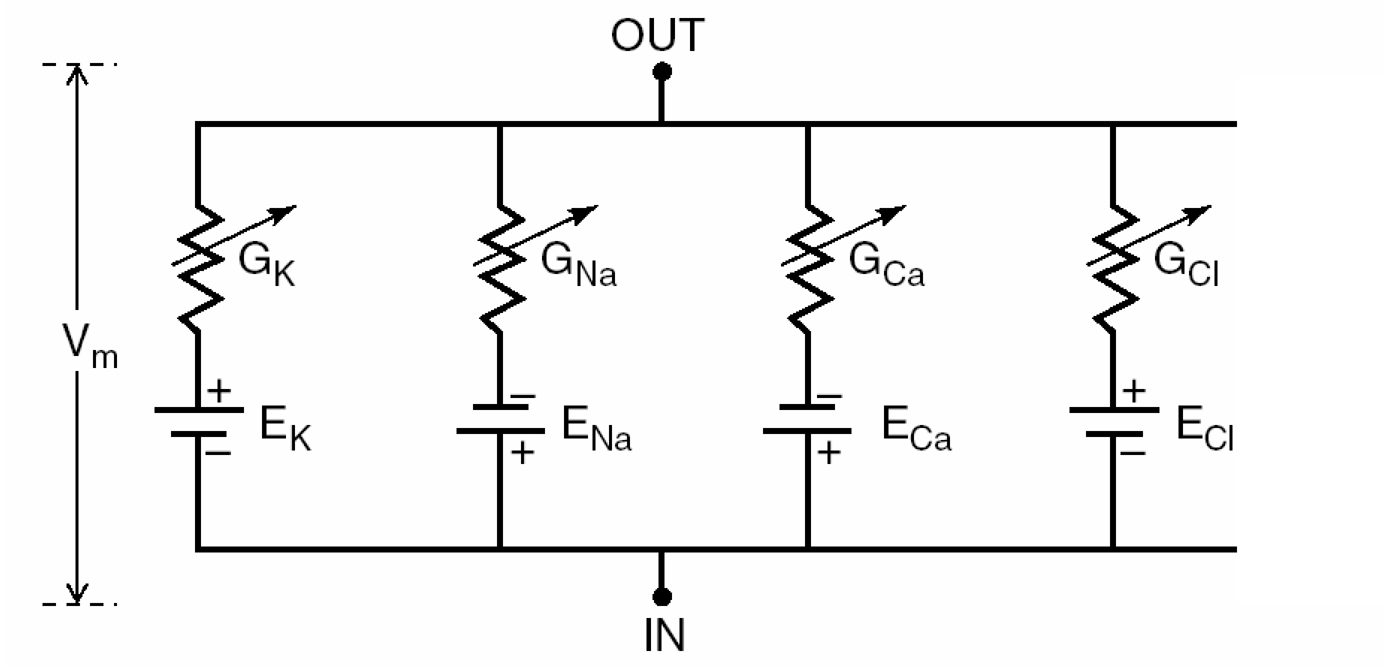


- Active balance between pumping and transport for Na^+ and K^+
- Cl^- configuration follows potential set by other system.

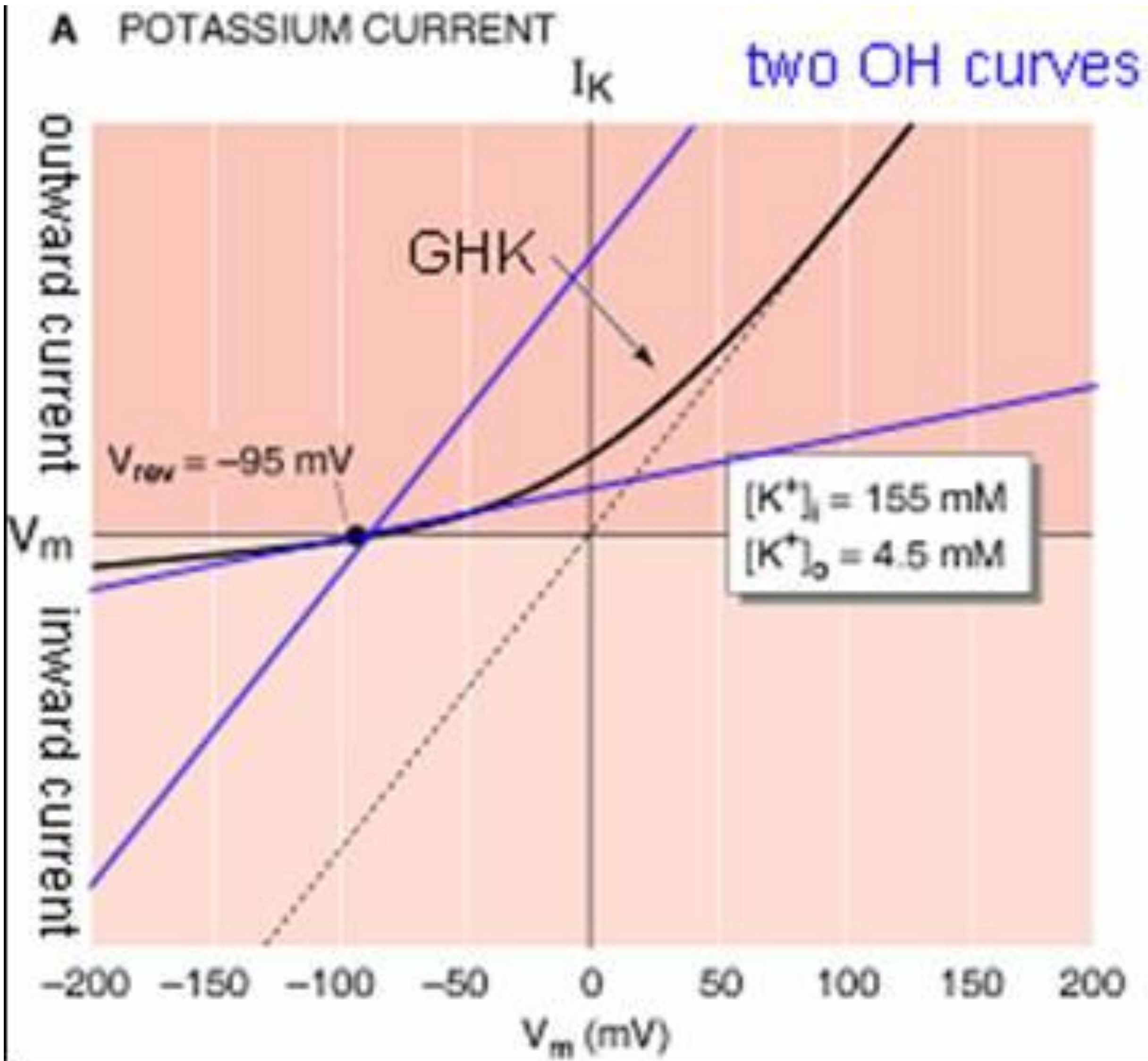
Nomenclature – version 1




inside
Vm
RNa =1/gNa
INa
IK
RK =1/gK
RCl =1/gCl
+
VNa Nernst
VK Nernst
VCl Nernst
-
outside


OUT
Vm
GK
GNa
GCa
GCl
EK
ENa
ECa
ECl
IN

GHK vs. Ohmic



A POTASSIUM CURRENT
I_K
two OH curves
GHK
outward current
inward current
V_m
V_{rev} = −95 mV
$[K^+]_i$ = 155 mM
$[K^+]_o$ = 4.5 mM
−200 −150 −100 −50 0 50 100 150 200
V_m (mV)